5.1 Analyzing and **Measuring Motion**



Experienced mountain bikers develop a sense of speed. They can react to bumps and turns almost automatically. However, it takes a lot of practice to develop the biking skills shown in Figure 5.1. Engineers who design cars, airplanes, and spacecraft need a different set of skills to do their job. Precisely measuring and calculating motion are critical when engineers are designing technologically advanced vehicles. However, engineers and physicists must all start at the same place — learning about the basics of motion, as you are about to do.

Scalars and Vectors

To communicate about motion, you need some basic information about the quantities used to describe motion. The following example will give you some insights into these quantities. A classmate invites a few friends over after school to study for an exam. He tells the group, "I live 1.2 km from school. If you walk at 4 km/h, it will take you only about 15 min to get there." Could you find the way to your classmate's house? He told you how far (distance) he lives from school and how long (time) it will take to get there if you walk at a reasonable pace (speed). However, you still do not know the direction in which to walk to his house.

Quantities that describe magnitude (size or amount) but not direction are called scalar quantities or scalars. The quantities that the student used to give instructions — speed, distance, and time — are scalar quantities. Information about direction is missing. Quantities that do include direction as well as magnitude are called vector quantities or vectors. Some examples of vector quantities are velocity, displacement, and position. To distinguish between these quantities, symbols for vectors are written with arrows above them while symbols for scalars are not. For example, the symbol for velocity is \vec{v} and the symbol for speed is v.

Defining Distance, Displacement, and Position

In the example above, the student told his classmates that he lived 1.2 km from the school. He was telling them the distance between the school and his house. Distance is a scalar quantity that describes the length of a path between two points or locations. The symbol for distance is Δd . The symbol in front of the d is the Greek letter delta. Physicists and mathematicians often use delta to mean a difference or change. In the case of distance, the delta means the change in the location of an object when it moves from one point to another.

In Figure 5.2, a student is giving instructions for going to his house 1.2 km away. If he was pointing, he would be providing a direction. By providing a direction, he would be describing the displacement from the school to his house. Displacement is a vector quantity. Displacement describes the straightline distance from one point to another as well as the direction. The symbol for displacement is Δd .



Figure 5.1 When you look at this picture, imagine how it would feel to grip the handle bars tightly as you soar into the air.



Figure 5.2 When you are telling someone how to go to your house, you usually point toward your house. Without realizing it, you are describing a vector quantity.

The delta in the symbol for displacement means the change in the position of an object when it is displaced. Position is a vector quantity that describes a specific point relative to a reference point. You can choose any point as a reference. If you use a co-ordinate system, the logical reference point would be point (0,0) where the axes cross. The symbol for position is \bar{d} . Displacement is defined mathematically as the difference between two positions, as shown in the following formula. The relationship between displacement vectors and position vectors is shown in Figure 5.3.

Displacement

 $\Delta \vec{d} = \vec{d}_2 - \vec{d}_1$

where $\Delta \vec{d}$ is displacement

 \vec{d}_1 is the initial or starting position

 \vec{d}_2 is the final or ending position

The SI unit for position and displacement is metres, m

To develop a clear understanding of the difference between distance and displacement, examine the map in Figure 5.4. The displacement from Edmonton to Peace River is shown by the arrow. To avoid clutter on the map, no reference point is chosen and position vectors are not included. The distance along the displacement arrow is approximately 365 km. The direction that the arrow points is about 40° west of north. That means that if you pointed an arrow north and then rotated it through an angle of 40° toward the west, you would be pointing in the direction of the displacement on the map. You would describe the displacement from Edmonton to Peace River as $\Delta \vec{d} = 365 \text{ km}[\text{N}40^{\circ}\text{W}].$

If you were to drive from Edmonton to Peace River, you could not drive in a straight line. You would follow the highways shown by the heavy line on the map. The distance along the highway (path between Edmonton and Peace River) is approximately 485 km. This distance is completely described by writing $\Delta d = 485$ km. No direction is involved. Notice that the distance is longer than the magnitude of the displacement. You can practise your skills in working with distance and displacement by completing the following activity.

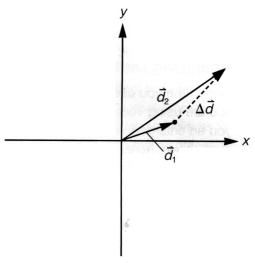


Figure 5.3 To graphically determine a displacement vector, start at the tip of the initial position vector and draw an arrow to the tip of the final position vector.

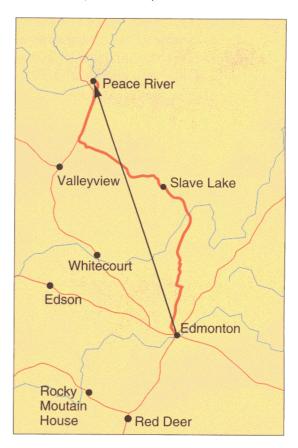


Figure 5.4 The distance along the highway from Edmonton to Peace River is longer than the magnitude of the displacement.

DidYou**Know**?

Measurement, communication, and precision are essential to science and the mathematics used in calculations for science. Confusion can lead to big and expensive problems. In 1999, NASA's Mars Climate Orbiter space probe disappeared. It probably burned up in the Martian atmosphere. Several engineering groups had worked on the probe, which cost more than \$300 million to design, construct, and launch. An investigation later found that one group had used SI units, such as metres and kilograms. Another group had assumed that data were being recorded in feet, inches, and pounds. These units are commonly used in the U.S. As a result, the computers on the probe made errors in the calculations for putting it into orbit.





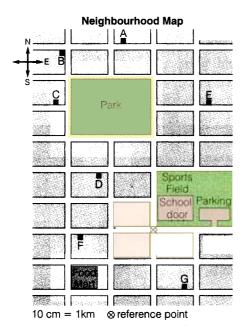
Analyzing Distance and Displacement

Every time you use a map or give directions, you are using vectors. Your teacher will give you an enlarged version of the neighbourhood map shown here. You will use the map for this activity. You will first determine the distance and displacement between several pairs of points. Then you will learn how to add vectors and subtract vectors graphically.

A length of 10 cm on the map you will use represents 1 km in actual distance in the neighbourhood. Use a ruler to measure lengths on the map. You can then find the actual distances by multiplying the measured map distances in centimetres by 1 km/10 cm. Use a protractor to determine directions. Use the intersection beside the school as your reference point when drawing position vectors. This intersection is marked with a ⊗ on your map. The first letter of a student's name is used to label his or her house on the map.

Materials

map ruler protractor pencil paper



Procedure

Distance versus displacement

- 1. Cate walks to Allison's house and they then walk to school together.
 - (a) Determine the total distance that Cate walks.
 - (b) Determine the total distance that Allison walks.

- (c) What is the displacement from Cate's house to Allison's house?
- (d) What is the displacement from Allison's house to the school?
- (e) What is the displacement from Cate's house to the school?
- 2. Esra and Donita go to the Food Mart after school to pick up some snacks. They then go to Donita's house to study for an exam. Before dinner. Esra walks home.
 - (a) Determine the total distance that Esra walks.
 - (b) Determine the total distance that Donita walks.
 - (c) What is the displacement from the school to Esra's house?
 - (d) What is the displacement from the school to Allison's house?

Adding Vectors

3. To add vectors, you place the tail of the second vector on the tip of the first vector. You then draw a vector from the tail of the first vector to the tip of the second vector. Without realizing it, you performed vector addition in parts (c), (d), and (e) of step 1 above. The mathematical expression for the addition of these vectors is:

$$\Delta \vec{d}_{\text{C to A}} + \Delta \vec{d}_{\text{A to S}} = \Delta \vec{d}_{\text{C to S}}$$

Examine your answers to step 1 (c), (d), and (e) and relate your process to the equation above.

4. Determine the displacement from Brad's house to the school. You can do this by adding the displacement vector from Brad's house to Frieda's house to the displacement vector from Frieda's house to the school. The mathematical statement is:

$$\Delta \vec{d}_{B \text{ to } F} + \Delta \vec{d}_{F \text{ to } S} = \Delta \vec{d}_{B \text{ to } S}$$

5. Find the sum $\Delta \vec{d}_{x \text{ to } z}$ of the following vectors.

$$\Delta \vec{d}_{x \text{ to } y} = 4.5 \text{ cm } [E30^{\circ}S]$$

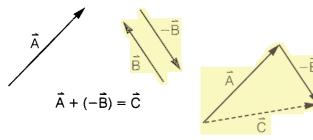
$$\Delta \vec{d}_{y \text{ to z}} = 6.3 \text{ cm } [E60^{\circ}N]$$

Subtracting Vectors

6. You learned that the definition of displacement is given by the equation:

$$\Delta \vec{d}_{1\to 2} = \vec{d}_2 - \vec{d}_1$$

To find a displacement vector from two known position vectors, you would have to subtract vectors. The graphical method for subtracting vectors is shown below.



To subtract \overline{B} from \overline{A} , define the negative of \overline{B} by drawing a line the same length as \overline{B} but pointed in the opposite direction. Add $-\overline{B}$ to \overline{A} by drawing a vector from the tail of \overline{A} to the tip of $-\overline{B}$. This new vector is the difference vector, or $\overline{C} = \overline{A} - \overline{B}$. Study the method until you can apply it to the map.

7. Use the method shown here to find the displacement from Frieda's house to Giesela's house. First, draw position vectors from the reference point near the school to Frieda's house and Giesela's house. Label them \vec{d}_F and \vec{d}_G . Subtract these position vectors as shown in the equation to find the displacement vector.

$$\Delta \vec{d}_{\mathsf{F} \to \mathsf{G}} = \vec{d}_{\mathsf{G}} - \vec{d}_{\mathsf{F}}$$

8. Use graphical vector subtraction to find the displacement from Giesela's house to the Food Mart.

What Did You Find Out?

- What did you observe about the relative lengths of distances and the magnitude of displacement vectors? Write a general statement about your conclusion.
- 2. Write a detailed method for adding vectors.
- 3. Write a detailed method for subtracting vectors.

Calculating Distance and Displacement

In the previous Find Out Activity, you added vectors in two dimensions graphically. In this textbook, you will perform mathematical calculations on vectors only in one dimension or along a straight line. The direction of the vectors will be indicated by compass directions (N, S, E, or W) or by positive and negative signs (+ or -). For example, Figure 5.5 shows you how to determine the displacement vector, $\Delta \vec{d}$, given two position vectors, \vec{d}_1 and \vec{d}_2 . When you subtract a vector, you point it in the opposite direction but keep the length the same. Because you will be working in only one dimension, the magnitudes of the velocity vectors can be added and subtracted algebraically. The Model Problem and Practice Problems that follow will help you add and subtract vectors in one dimension.

m[E] 50 m[E]
$$\vec{d}_1$$
 \vec{d}_2 50 m[E] \vec{d}_2 50 m[E] \vec{d}_3 \vec{d}_4 \vec{d}_5 \vec{d}_7 \vec{d}_8 \vec{d}_9 \vec{d}_9

Figure 5.5 Position \vec{d}_1 is the point at which the runner is 10 m east of the reference point or 0.0 m. Position \vec{d}_2 is 50 m east of the reference. The runner's displacement, $\Delta \vec{d}$, is 40 m[E].

Model Problem 1

Part A

Find the displacement vector from position A to position B if $\vec{d}_A = +3.5$ cm and $\vec{d}_B = +5.7$ cm.

Given

position A, $\vec{d}_A = +3.5$ cm position B, $\vec{d}_B = +5.7$ cm

Required

displacement, $\Delta \vec{d}_{A\to B}$

Analysis

The two position vectors are given so you can use the definition of displacement, $\Delta \vec{d} = \vec{d}_2 - \vec{d}_1$. In one dimension, the magnitudes of vectors can be added or subtracted algebraically.

Solution

$$\Delta \vec{d}_{\text{A to B}} = \vec{d}_{\text{B}} - \vec{d}_{\text{A}}$$

$$\Delta \vec{d}_{\text{A to B}} = +5.7 \text{ cm} - (+ 5 \text{ cm})$$

$$\Delta \vec{d}_{\text{A to B}} = +2.2 \text{ cm}$$

Paraphrase

The displacement is 2.2 cm in the positive direction.

Part B

Find the displacement vector from position X to position Y if $\vec{d}_X = 6.9 \text{ m}[\text{E}]$ and $\vec{d}_Y = 8.2 \text{ m}[\text{W}]$.

Given

position X, $\vec{d}_X = 6.9 \text{ m[E]}$ position Y, $\vec{d}_Y = 8.2 \text{ m[W]}$

Required

displacement, $\Delta \bar{d}_{X\to Y}$

Analysis

The direction "west" is equivalent to the negative of "east." Therefore, write position Y as $\vec{d}_Y = -8.2$ m[E]. Now you can use the definition of displacement, $\Delta \vec{d} = \vec{d}_2 - \vec{d}_1$. In one dimension, the magnitudes of vectors can be added or subtracted algebraically.

Solution

$$\Delta \vec{d}_{X \to Y} = \vec{d}_Y - \vec{d}_X$$

$$\Delta \vec{d}_{X \to Y} = -8.2 \text{ m[E]} - (6.9 \text{ m[E]})$$

$$\Delta \vec{d}_{X \to Y} = -15.1 \text{ m[E]}$$

$$\Delta \vec{d}_{X \to Y} \approx 15 \text{ m[W]}$$

PROBLEM TIP

The symbol \approx means "approximately equal." When rounding an answer to the correct number of significant digits, physicists and mathematicians often write the answer with a \approx .

Paraphrase

The displacement is negative 15 m east, which is the same as a positive 15 m west.

Part C

Alonzo walks 0.64 km north and then walks 1.76 km south. What was Alonzo's total displacement?

Given

first displacement, $\Delta \vec{d}_1 = 0.64 \text{ km}[N]$ second displacement, $\Delta \vec{d}_2 = 1.76 \text{ km}[S]$

Required

total displacement, $\Delta \vec{d}_{\text{total}}$

Analysis

The total displacement is the vector sum of the two displacements. Since the vectors are along a straight line, the magnitudes can be added algebraically. Since south is the negative of north, the second vector can be written, $\Delta \vec{d}_2 = -1.76$ km [N].

Solution

The vectors are known, so you can just add them.

$$\Delta \vec{d}_{\text{total}} = \Delta \vec{d}_1 + \Delta \vec{d}_2$$

$$\Delta \vec{d}_{\text{total}} = 0.64 \text{ km[N]} + (-1.76 \text{ km[N]})$$

$$\Delta \vec{d}_{\text{total}} = -1.12 \text{ km[N]}$$

$$\Delta \vec{d}_{\text{total}} \approx 1.1 \text{ km[S]}$$

Paraphrase

The total displacement is negative 1.1 km north, which is the same as positive 1.1 km south.

Practice Problems

- 1. Find the displacement from position 1, $\vec{d}_1 = +45$ cm, to position 2, $\bar{d}_2 = +33$ cm.
- 2. Matthew is standing at a position described by $\bar{d}_{\rm M} = +73$ m. Jason is standing at $\bar{d}_1 = -18$ m. What is the displacement from Matthew to Jason?
- 3. On a chessboard, a king sits at a position +12 cm from the lower left corner. A knight is +5.0 cm from the same corner along the same straight line. What is the displacement from the king to the knight?
- 4. Grant is standing at position $\bar{d}_G = 45 \text{ m[W]}$ and Serina is at position $\overline{d}_S = 22 \text{ m}[W]$. What is the displacement from Grant to Serina?
- 5. Alex walks 1.4 km west from the school. Kendra walks 0.45 km east. What is the displacement from Alex to Kendra?
- 6. Brooks is 111 km south of Hanna. Killam is 122 km north of Hanna. What is the displacement from Brooks to Killam?
- 7. Raja drives east from Olds to Trochu, a distance of 59 km. He then drives west to Sundre, a distance of 97 km. What distance did Raja drive? What was his total displacement?
- 8. You have learned how to take steps that are exactly 1 m long. You take 12 steps north, then 15 steps north, 35 steps south, 11 steps south, then 16 steps north. What distance have you walked? What is your total displacement?
- 9. Jennifer throws a softball 7.8 m east to Sasha. Sasha throws it 9.5 m east to Clair. Clair throws it 8.2 m west to Brianna. Brianna throws it 9.1 m west to Kerry. What distance did the softball travel? What was the total displacement of the softball?







Speed and Velocity

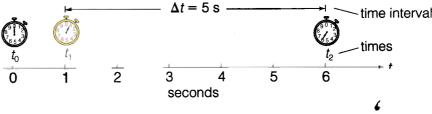
The terms "speed" and "velocity" are probably quite familiar to you. Think of a weather report in which the meteorologist is reporting wind velocities. Think of auto races in which cars run at speeds of over 200 km/h. Have you ever thought about the difference between speed and velocity? As you read at the beginning of this section, speed is a scalar and velocity is a vector. Speed is related to distance and velocity is related to displacement. Both quantities involve time.

If you were going to determine a sprinter's speed or velocity, you would probably start a stopwatch the instant the sprinter left the starting line. However, you might want to know how fast a certain sprinter was running during the first half and last half of a race. Assume that the race is a 50 m dash. You would start the stopwatch when the race started. Then you would check your stopwatch when the sprinter reached the 25 m line and again at the end of the race. You would have measured two time intervals. How does a time interval differ from a time?



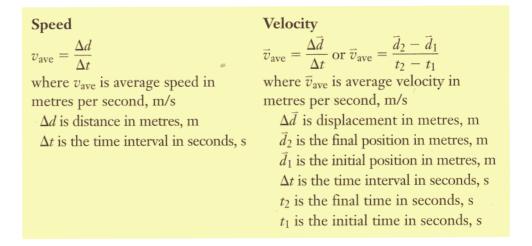
Time (t) is a point in time as it relates to your reference or zero time. As shown in Figure 5.6, a time interval (Δt) is the difference between two times. Both quantities are scalars. When you calculate speed or velocity, you must specify the time interval during which you are determining the speed or velocity. The mathematical formula for time interval is given below.

Figure 5.6 You usually choose time zero as the time at which you start your stopwatch. All times after that are determined in relation to time zero.



Time Interval $\Delta t = t_2 - t_1$ where Δt is time interval t_1 is the initial or starting time t_2 is the final or ending time The SI unit for time and time intervals is seconds, s.

Now you are ready to define speed and velocity mathematically. Speed is the distance travelled by an object during a given time interval divided by the time interval. Velocity is the displacement of an object during a time interval divided by the time interval. The direction of the velocity is the same as the direction of the displacement. Since the speed and velocity might change during an interval of time, the formulas below represent average speed and average velocity. The formulas look very similar, but an example will reveal some slight differences. Apply the formulas below to a trip from Edmonton to Peace River.



Imagine that your family was driving from Edmonton to Peace River. It is a long trip so you stopped once for a meal and two other times for snacks. The entire trip took 7.5 h. As you read previously, the distance along the highway is 485 km and the displacement is 365 km[N40°W]. You could calculate the speed and velocity as shown on the next page.

Speed

$$v_{\text{ave}} = \frac{\Delta d}{\Delta t}$$
 $v_{\text{ave}} = \frac{485 \text{ km}}{7.5 \text{ h}}$

$$v_{\text{ave}} = 7.5 \text{ h}$$

$$v_{\text{ave}} = 64.667 \frac{\text{km}}{\text{h}}$$

$$v_{\rm ave} \approx 65 \, \frac{\rm km}{1}$$

$$v_{\rm ave} \approx 65 \, \frac{\rm km}{\rm h}$$

Velocity

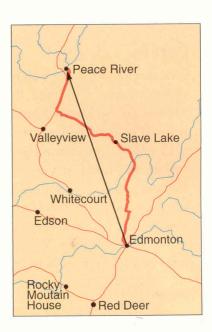
$$\vec{v}_{\mathrm{ave}} = \frac{\Delta \vec{d}}{\Delta t}$$

$$\vec{v}_{\text{ave}} = \frac{365 \text{ km}[\text{N}40^{\circ}\text{W}]}{7.5 \text{ h}}$$

$$\vec{v}_{\text{ave}} = 48.667 \, \frac{\text{km}}{\text{h}} \, [\text{N}40^{\circ} \text{W}]$$

$$\vec{v}_{\rm ave} \approx 49 \, \frac{\rm km}{\rm h} \, [{
m N}40^{\circ}{
m W}]$$

Although the values for speed and velocity describe the same trip, they look quite different. The reason for the difference lies in the definitions of distance and displacement. Remember that displacement is always measured along a straight line joining the initial and final positions. Distance, however, is measured along the actual path taken. The time intervals are the same. Study the Model Problems below and complete the Practice Problems on pages 184 and 185 to sharpen your problem-solving skills.



Math SCONNECT

When speed is given in kilometres per hour, you can convert to metres per second. Convert km/h to SI units by multiplying your quantity by the ratio of metres to kilometres and by the ratio of hours to seconds, as shown here. For example, if you were given the value of 55 km/h and wanted to convert it to m/s, you would do the following.

$$\left(\frac{55 \text{ km}}{\text{M}}\right) \left(\frac{1000 \text{ m}}{1 \text{ km}}\right) \left(\frac{1 \text{ M}}{3600 \text{ s}}\right) = \frac{(55)(1000)}{3600} \frac{\text{m}}{\text{s}} = 15.28 \frac{\text{m}}{\text{s}}$$

When the units cancel properly, you know that your calculations are correct.

Next, you should report the answer in the correct number of significant digits. Since 55 km/h has two significant digits, the answer should also have two significant digits or 15 m/s.

For practice, convert 18 km/h to units of m/s.

Model Problem 2

Part A

A car travelled a distance of 550 m in a time interval of 35 s. What was the speed of the car?

Given

distance travelled by car, $\Delta d = 550 \text{ m}$ time interval during which the car was observed, $\Delta t = 35 \text{ s}$

Required

speed of the car, v

Analysis

The data given are time interval and distance. Therefore, use the formula that involves distance and time

interval,
$$v = \frac{\Delta d}{\Delta t}$$
.

Solution

$$v = \frac{\Delta d}{\Delta t}$$

$$v = \frac{550 \text{ m}}{35 \text{ s}}$$

$$v = 15.714 \frac{\text{m}}{\text{s}}$$

$$v \approx 16 \frac{\text{m}}{\text{s}}$$

Paraphrase

The car was travelling at a speed of 16 m/s.

Part B

Two trainers with stopwatches are timing a runner who is training for a race. Both timers start their stopwatches when the runner leaves the starting point. The first trainer is standing at a position that is 12 m[S] of the starting point and the second trainer is standing at a position 65 m[S] of the starting point. Each trainer stops her stopwatch when the runner passes her. The first trainer's stopwatch reads 1.6 s and the second trainer's stopwatch reads 8.7 s. What was the athlete's velocity while racing between the trainers?

Given

initial position, $\bar{d}_1 = 12 \text{ m[S]}$ final position, $\bar{d}_2 = 65 \text{ m[S]}$ initial time, $t_1 = 1.6 \text{ s}$ final time, $t_2 = 8.7 \text{ s}$

Required

Velocity of the runner, \bar{v}

Analysis

The problem gives the positions of the trainers. They stopped their stopwatches when the runner was at those positions. The trainers also started their stopwatches when the runner was at position zero. Therefore, the readings on their stopwatches are the times that the runner passed each position. Use the formula for velocity that involves positions $\frac{1}{dx} = \frac{1}{dx}$

and times,
$$\overline{v} = \frac{\overline{d}_2 - \overline{d}_1}{t_2 - t_1}$$

Solution

$$\bar{v} = \frac{\bar{d}_2 - \bar{d}_1}{t_2 - t_1}$$

$$\bar{v} = \frac{65 \text{ m[S]} - 12 \text{ m[S]}}{8.7 \text{ s} - 1.6 \text{ s}}$$

$$\bar{v} = \frac{53 \text{ m[S]}}{7.3 \text{ s}}$$

$$\bar{v} = 7.26 \frac{\text{m}}{\text{s}}[\text{S}]$$

$$\bar{v} \approx 7.3 \frac{\text{m}}{\text{s}}[\text{S}]$$

Paraphrase

The athlete was running at a velocity of 7.3 m/s[S].



Practice Problems

All of your answers should be in SI units. Read the Math Connect on the previous page to review the method for converting km/h to m/s.

Use the method that involves distances or displacements and time intervals to complete problems 10 through 14.

- 10. A stunt bicycle rider goes 39 m in 3.0 s. How fast is the cyclist riding?
- 11. A skier goes 148 m[W] in 5.50 s. What is the skier's velocity?
- 12. A jet plane travels from Calgary to Winnipeg, a distance of 1358 km, in 2 h and 45 min. Determine the speed of the jet plane in metres per second. (Hint: There are 1000 m in 1 km, 60 s in 1 min, and 3600 s in 1 h.)
- **13.** A cheetah runs at a velocity of 29 m/s[N]. If it runs for 8.4 s, what is its displacement?
- 14. You and your family are driving to your grandparents' home, which is 95 km away. If you drive at an average speed of 85 km/h, how long will it take you to get there?

Use the method that involves positions and times to complete problems 15 through 17.

- 15. A 100 m track is marked off in metres. When a sprinter leaves the starting line, timers are started. The sprinter passes the 12 m[E] mark at 1.8 s and passes the 56 m[E] mark at 6.7 s. What was the sprinter's velocity between those two positions?
- 16. The fence posts around a large pasture are 2.5 m apart. A horse starts running west beside the fence. When the horse passes the fifth fence post, your second hand is on the 9.0 s mark. When the horse passes the fourteenth fence post, the second hand is on 11.5 s. What is the horse's velocity?
- 17. On some highways, exit signs are numbered according to the number of kilometres the exit is from the place where the highway originated. If you are driving south and pass exit 35 at 2:15 p.m. and then you pass exit 116 at 3:09 p.m., what is your velocity?

Analyze each problem to decide which method applies to problems 18 through 22.

- 18. A cougar can leap 11 m horizontally. If it spends 1.8 s in the air, what was its speed?
- 19. If a maglev (magnetic levitation) train ran between Edmonton and Calgary, a distance of 295 km, it could make the trip in about 0.75 h (three quarters of an hour). What would be the speed of the maglev train?
- 20. A car and driver leave Whitecourt at noon. The car passes through Valleyview at 2:15 p.m. and reaches Grande Prairie at 3:45 p.m., including time to stop for a meal. Valleyview is 168 km from Whitecourt and Grande Prairie is 279 km from Whitecourt. What was the average speed of the car between Valleyview and Grande Prairie?
- 21. Some teenagers are training their dog to carry messages between the store where one teen works and the restaurant where the other works. They want to find out how long it will take the dog to make the trip. They measure the displacement between the store and the restaurant and find that it is 1.2 km[W]. While training the dog, they time it with a stopwatch and find the time to be 14 min. What was the dog's velocity?
- 22. If you walk to school at a speed of 1.2 m/s and it takes you 18 min to reach the school, what is the distance from your home to the school?

You have learned the definitions for speed and velocity and have practised calculating speed. You can now start collecting your own data. In the next activity, observe the motion of a laboratory cart and measure the distance that it travels over several seconds. Determine its velocity for a few different time intervals. Draw conclusions about the motion of the cart.







Measuring Velocity in One Dimension

If an object is travelling slowly in a straight line, you can gather data about the object's motion using simple tools.

Materials

laboratory cart or friction block stopwatch paper tape masking tape two coloured pencils or pens string or wire metre stick or measuring tape

Procedure • Performing and Recording

- Stretch a paper tape along a smooth surface, such as a table top, and tape it down using the masking tape.
- **2.** Attach a string or wire to a laboratory cart (or friction block).



- **3.** The second team member will watch the stopwatch and count each second aloud.
- 4. Trial 1: Have the third team member pull the cart at a constant slow speed beside the paper tape. Walk along, marking the cart's position with a coloured pencil on the tape each time the team member counts the seconds. Do this for 6 s. (Hint: Start pulling the cart before you start taking data.)
- 5. Trial 2: Repeat Trial 1. This time, try to move the cart at a faster constant speed. Use a different coloured pencil to mark its position.

6. Let the point at which you started measuring represent a position of zero. Now measure from the starting point to each successive dot. Record your observations in a table, such as the one below.

Trial 1		Trial 2	
Time (s)	Position (m)	Time (s)	Position (m)
0.0	0.0	0.0	0.0

- 7. For each of the two trials, calculate the velocity for every time interval. (For example, calculate the velocity between 0.0 s and 1.0 s, between 1.0 s and 2.0 s, and so on.)
- 8. Calculate the average velocity for each of the two trials. You can find the average velocity by adding the velocities of all the time intervals, then dividing the sum by the number of time intervals. (This method is correct only if the time intervals were all the same. For this activity, each time interval should be 1 s.)
- For both trials, calculate the velocity for the full 6 s. (Note: Save your data for use in another activity that you will carry out later.)

What Did You Find Out? • Analyzing and Interpreting

- Was the velocity constant for Trial 1? for Trial 2? Explain how you can determine whether the velocity was constant.
- 2. How does the average velocity for Trial 1 and Trial 2 compare to the velocity that you calculated for the full 6 s for the given trial?
- 3. Was it easier to maintain a constant velocity for a slow speed (Trial 1) or a greater speed (Trial 2)?

Career SCONNECT

Teri MacDonald-Cadieux is a professional road-racing driver. Her job demands special physical and mental qualities. She must pilot her car around complex, twisting courses. Her first racing experience came at the age of 11, behind the wheel of a go-cart. Teri's worst moment in racing came in 1997. Her car crashed into a wall at over 200 km/h. It rolled several times before coming to a stop. In the accident, Teri fractured a vertebra in her neck. Within three months, however, she was back in the driver's seat.

Teri's love of driving goes beyond her role as a driver. She teaches race-driving techniques, runs car-care clinics, and is a certified child safety seat inspector. As well, she presents a television feature called "Car Sense." In this feature, she aims to demystify the automobile.

List the qualities that you think would be needed in Teri's work In what other careers might these qualities be valuable?



Teri MacDonald-Cadieux

Graphing Velocity

You often understand a topic better if you approach it in several different ways. Some people learn from visualization. Working with data tables, pictures, and graphs might help you develop a deeper understanding of the relationships among position, displacement, time, and velocity. Examine Figure 5.7 and analyze how the motion of the sprinters matches the graphs below them.

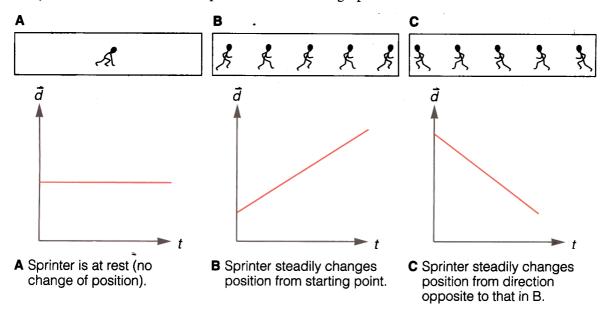
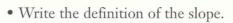


Figure 5.7 The zero position is on the left in the illustrations of the sprinters at the top of the figure. The motion takes place along the position axis, which is the vertical axis on the graphs.

All of the motion of the sprinters in Figure 5.7 takes place along a straight line. In the graphs, that line is the \bar{d} axis or vertical axis. The time between each image of the sprinter is 1 s. In part A of Figure 5.7, the sprinter is in the starting position. Thus the position, \overline{d} , is the same at all times. In part B, you can see that the sprinter is running at a constant velocity because the distance between the images is the same. The sprinter is starting near the assigned zero position and moving away from it. Therefore, the line in the graph goes up — or farther from the zero position — as time passes. In part C, the sprinter starts at a greater

displacement from zero and runs toward the zero position. Notice that the line on the graph starts at a large value of d and goes down toward the zero position as time passes. In each case, the velocity of the sprinter is constant. Physicists call this type of motion uniform motion because there is no change in velocity.

You can directly determine the velocity of an object that is moving with uniform motion from a graph of position versus time. To start developing the method, study Figure 5.8. Recall from mathematics that the slope of a straight line is defined as the "rise" over the "run." In Figure 5.8, you can see that the "rise" on a position versus time graph is the displacement of the object. The "run" is the time interval during which the motion occurred. Examine the figure as you read through the following steps.



slope =
$$\frac{\text{rise}}{\text{run}}$$

slope =
$$\frac{\Delta \vec{d}}{\Delta t}$$

$$\vec{v} = \frac{\Delta \vec{d}}{\Delta t}$$
$$\vec{v} = \text{slope}$$

To learn how to use the relationship between velocity and slope, examine the data table and graph, then follow the calculations below. The data in Table 5.1 were used to plot the points on the graph in Figure 5.9.

à rise! run

Figure 5.8 The velocity of an object is the slope of the graph of position versus time.

Table 5.1 Experimental data

Time (s)	Position (m[E])
0.0	0.0
1.5	12
3.0	24
4.5	36
6.0	48
7.5	60

Choose the first point (0.0 s, 0.0 m) and the last point (7.5 s, 60 m).

slope =
$$\frac{\Delta \vec{d}}{\Delta t}$$

slope = $\frac{60 \text{ m[E]} - 0.0 \text{ m[E]}}{7.5 \text{ s} - 0.0 \text{ s}}$

slope =
$$\frac{60 \text{ m[E]}}{7.5 \text{ s}}$$

slope =
$$8.0 \frac{\text{m}}{\text{s}} [\text{E}]$$

$$\vec{v} = 8.0 \, \frac{\text{m}}{\text{s}} [\text{E}]$$

Choose the second point (1.5 s, 12 m) and the third point (3.0 s, 24 m).

slope =
$$\frac{\Delta \vec{d}}{\Delta t}$$

slope =
$$\frac{24 \text{ m[E]} - 12 \text{ m[E]}}{3.0 \text{ s} - 1.5 \text{ s}}$$

slope =
$$\frac{12 \text{ m[E]}}{1.5 \text{ s}}$$

slope =
$$8.0 \frac{\text{m}}{\text{s}} [\text{E}]$$

$$\vec{v} = 8.0 \, \frac{\text{m}}{\text{s}} [\text{E}]$$

Choose the third point (3.0 s, 24 m) and the fifth point (6.0 s, 48 m).

slope =
$$\frac{\Delta \vec{d}}{\Delta t}$$

slope =
$$\frac{48 \text{ m[E]} - 24 \text{ m[E]}}{6.0 \text{ s} - 3.0 \text{ s}}$$

slope =
$$\frac{24 \text{ m[E]}}{3.0 \text{ s}}$$

slope =
$$8.0 \frac{\text{m}}{\text{s}} [\text{E}]$$

$$\overline{v} = 8.0 \, \frac{\text{m}}{\text{s}} [\text{E}]$$



Figure 5.9 You can pick any two points on the straight line to determine the slope and thus the velocity of the object.

Notice that the velocity is the same in each case. This result supports the statement that when a position versus time graph is a straight line, the velocity is constant.

Data points that lie precisely on a straight line, such as those in Figure 5.9, are unusual. More frequently, you will detect small measurement errors. For example, the time it takes to push the button on a stopwatch will vary with each measurement. Also, it is difficult to read a metre stick with precision. For these reasons, graphs plotted from experimental data usually look more like the graph in Figure 5.10. You can get the best results from these data by drawing a "line of best fit" through the points, as shown

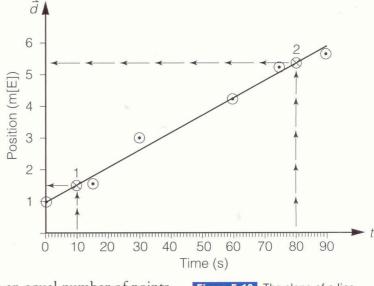


Figure 5.10 The slope of a line of best fit is a more accurate calculation of velocity than the velocity calculated from two data points.

in the figure. To draw a line of best fit, try to leave an equal number of points above and below the line. When you calculate the slope of the line, do not use any of the actual data points because most of them do not lie directly on the line. For the most accurate calculation, choose two points on the line that are near the opposite ends of the line. Calculate the slope from those points. For example, as shown in Figure 5.10, you could choose 10 s and 80 s. Start at those numbers on the time axis and go up to the line. Mark the point on the line with a symbol such as a cross. Then go left from those points to the position axis to find the value of the position that corresponds to each of the two times. The selected points on the line in Figure 5.10 have the co-ordinates (10 s, 1.6 m[E]) and (80 s, 5.3 m[E]). Calculate the slope as follows. Practise these techniques in the activity on the next page.

slope =
$$\frac{\vec{d}_2 - \vec{d}_1}{t_2 - t_1}$$

slope = $\frac{5.3 \text{ m[E]} - 1.6 \text{ m[E]}}{80 \text{ s} - 10 \text{ s}}$
slope = $\frac{3.7 \text{ m[E]}}{70 \text{ s}}$
slope = $0.05286 \frac{\text{m}}{\text{s}} \text{[E]}$
 $\vec{v} \approx 0.053 \frac{\text{m}}{\text{s}} \text{[E]}$

Defining Acceleration

How would it feel to drive the dragster shown in Figure 5.11? Some dragsters can go from a full stop to speeds of over 89 m/s (320 km/h) in less than 4 s. Imagine how the driver is pressed back against the seat. Even under normal driving conditions, you can feel a change in the motion of a car when you speed up, brake rapidly, or turn quickly. However, when a car is moving at a constant speed — uniform motion — you are almost unaware of any motion at all. What is unique about changes in velocity compared to constant velocity?



Figure 5.11 Try to imagine how the driver of this dragster feels as the speed increases rapidly.



Physicists define acceleration as any change in the velocity of an object during a time interval. The change might be an increase or decrease in the magnitude of the velocity or a change in the direction of the object. Since velocity is a vector quantity and acceleration is a change in velocity, acceleration is also a vector quantity.

To explain why you feel a change in motion, or an acceleration, think about the cause of acceleration. Imagine an ice surface that is so smooth that when a hockey puck slides across it, there is no friction. What would speed up, slow down, or change the direction of the hockey puck? You would have to hit it, or exert a force on it. When you feel a change in the direction of a car in which you are riding, you are actually feeling the force that is causing a change in the motion of your body. If no forces act on an object in uniform motion, the motion will not change.

Calculating Acceleration

The mathematical forms of the equations that describe acceleration are very similar to those for velocity, as shown on the next page. When you first look at the units, metres per second squared, they might seem strange. A brief analysis should clarify the meaning. When an object is accelerating, the velocity is changing. The numerical value of the acceleration states how much the velocity is changing. A value of 1.2 m/s² means that the velocity is changing by 1.2 m/s every second.

Graphing Position-Time Data

In this activity, you will use the graphical method for determining velocity from a position versus time plot.

Materials

data from Find Out Activity: Measuring Velocity in One Dimension graph paper

Procedure

- 1. Obtain your data tables from the Find Out Activity: "Measuring Velocity in One Dimension." Also, find and record the average velocities that you determined for the motion for the two trials in the activity.
- 2. On separate pieces of graph paper, make graphs of position on the vertical axis and time on the horizontal axis for each trial.
- 3. On each graph, draw a line of best fit.



4. Determine the slope of the line on each graph. Include units for the slope.

What Did You Find Out?

- 1. When you were collecting these data, you tried to achieve uniform motion. How well did you succeed in creating motion with a constant velocity? How does your graph support your answer?
- 2. How well does the slope of each graph agree with the average velocity that you calculated in the original activity? Why might you expect the values to agree?
- 3. Explain how you would use your graph to predict the displacement of the cart 10 s after you stopped taking data.
- 4. How would the graph appear if you had allowed the cart to slow down?

Acceleration

$$\vec{a}_{\text{ave}} = \frac{\Delta \vec{v}}{\Delta t} \text{ or } \vec{a}_{\text{ave}} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1}$$

where \vec{a}_{ave} is average acceleration in metres per second squared, m/s²

 $\Delta \bar{v}$ is the change in velocity in metres per second, m/s

 \vec{v}_2 is the final velocity in metres per second, m/s

 \vec{v}_1 is the initial velocity in metres per second, m/s Δt is the time interval in seconds, s

 t_2 is the final time in seconds, s

 t_1 is the initial time in seconds, s

Unit Analysis

$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$$

$$\frac{m}{s^2} = \frac{\frac{m}{s}}{s}$$

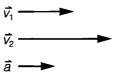
$$=\frac{\mathrm{m}}{\mathrm{s}}\div\frac{\mathrm{s}}{\mathrm{1}}$$

$$=\frac{\mathrm{m}}{\mathrm{s}}\times\frac{1}{\mathrm{s}}$$

$$=\frac{\mathrm{m}}{\mathrm{s}^2}$$

The direction of the acceleration is the same as the direction of the change in the velocity. To determine the direction of the acceleration from the initial and final velocities of an object, picture the direction in which you would have to push on the object in order to cause the observed change. The following examples will help you visualize the meaning of the direction of the acceleration.

- The initial velocity is in the positive direction. The final velocity is in the positive direction and the magnitude is larger. The acceleration is in the same direction as the initial velocity.
- The initial velocity is in the positive direction. The final velocity is in the positive direction, but the magnitude is smaller. The acceleration is in a direction opposite to the initial velocity.
- The initial velocity is in the positive direction. The final velocity is in the negative direction. The object slowed down, stopped, and began to move in the opposite direction. The acceleration is in a direction opposite to the initial velocity.
- The initial velocity is in the negative direction. The final velocity is in the negative direction, but the magnitude is smaller. The acceleration is in a direction opposite to the initial velocity.







When solving problems mathematically, the sign — positive or negative of the answer will tell you the correct direction of the acceleration. Study the following Model Problem, then complete the Practice Problems, After solving each problem, analyze the sign of the answer. Ensure that it gives the direction that you would expect. This method is one way to ensure that your answer is correct.

Model Problem 3

A naturalist observed a cheetah reach a speed of 19 m/s from a standing start in a period of 2.0 s. What was the cheetah's acceleration? Assume that the cheetah runs in a positive direction.

Given

Since the initial velocity is zero, the change in velocity is the same as the final velocity or

$$\Delta \overline{v} = +19 \, \frac{\mathrm{m}}{\mathrm{s}}$$

$$\Delta t = 2.0 \text{ s}$$

Required

acceleration, a

Analysis

The change in the velocity and the time interval are known, so you can use the definition for acceleration,

$$\vec{a}_{\text{ave}} = \frac{\Delta \vec{v}}{\Delta t}$$

Solution

$$\bar{a}_{\rm ave} = \frac{\Delta \vec{v}}{\Delta t}$$

$$\overline{a}_{ave} = \frac{+19 \frac{m}{s}}{2.0 s}$$

$$\bar{a}_{ave} = +9.5 \, \frac{\mathrm{m}}{\mathrm{s}^2}$$

Paraphrase

The cheetah's acceleration is 9.5 m/s2 in the positive direction.

Practice Problems

- 23. In a record-setting race, a dragster reached a velocity of 145.08 m/s in 4.48 s. What was the dragster's average acceleration? Assume that the direction of the velocity is positive.
- 24. A model rocket started from the ground and reached an upward velocity of 66 m/s in 5.0 s. What was the rocket's average acceleration? (Let the upward direction be positive.)
- 25. A student on a bicycle decided to determine his acceleration when coasting down a steep hill. The student started from a full stop and reached a velocity of 8.75 m/s in 3.8 s. What was his average acceleration? Assume that downhill is the positive direction.
- 26. A car enters a highway travelling 14 m/s[N]. After 5.5 s, the car reaches a velocity of 28 m/s[N]. What was the car's average acceleration?
- 27. A professional baseball pitcher pitches a ball, giving it a velocity of 45 m/s toward the batter. The batted ball has a velocity of 30 m/s toward the pitcher. Let the direction from the batter to the pitcher be the positive direction. If the change in velocity takes place over a period of 1.2 s, what was the average acceleration of the baseball?
- 28. A child rolled a ball up a hill. At time zero, the ball had a velocity of 1.8 m/s up the hill. After 6.5 sthe ball's velocity was 2.3 m/s down the hill. Let uphill be the positive direction. What was the average acceleration of the ball? What is the meaning of the sign of the acceleration?
- 29. Objects near Earth's surface fall with an acceleration of 9.81 m/s². If you dropped a rock from a cliff over a river, how fast would the rock be falling 4.1 s after you dropped it?



- **30.** The average acceleration of the space shuttle at takeoff is 29 m/s²[up]. What is the shuttle's velocity after 12 s? Let up be the positive direction.
- 31. A car is initially travelling at a velocity of 4.2 m/s[W]. If the car's average acceleration is 0.86 m/s²[W], how long will it take the car to reach a velocity of 9.6 m/s[W]?

Have you ever dreamed of becoming an astronaut? How do you make such a dream — any impossible-sounding dream come true? You could ask Julie Payette. On May 29, 1999, she became the eighth Canadian to fly into space, aboard the Space Shuttle Discovery. For Julie, this ten-day mission was just the latest stage in a long and varied journey that began in Montréal, Québec, where she was born in 1962. That journey has taken her to many different places in Europe, the United States, and Canada, in pursuit of her goals.

The first stop on Julie's quest was Atlantic College in Wales, U.K., where she won a scholarship to study at age 16. "Atlantic College helped open my mind and broaden my horizons. I met people from all over the world and shared incredible experiences." Julie returned to Montréal to earn a degree in engineering at McGill University. She then attained a Master's degree in computer processing design at the University of Toronto. This was followed by further work on computer voice recognition in Zurich, Switzerland, and at McGill.

Her astronaut training with the CSA (Canadian Space Agency) and NASA has taken her to Moose Jaw, Saskatchewan, to qualify as a pilot, and to Houston, Texas, to train for her role in the 1999 mission. Julie acted as the on-board "director" for the mission's space walks.



Julie Pavette

Julie Payette has a truly global perspective. As she says, "To work while orbiting Earth, to contribute to the pursuit of scientific knowledge, and to be able to see our world from above is an extraordinary privilege. From orbit, you can't see political borders, but in a truly global fashion, you can help monitor and preserve our beautiful planet — our only home — for everyone."

Graphing Accelerated Motion

How does a position versus time graph of accelerated motion differ from a graph of uniform motion? Examine the graphs in Figure 5.12 of sprinters speeding up and slowing down. Once again, the time between images is 1 s. You can tell that the sprinter in graph A is speeding up because the distance that the sprinter runs in one second becomes greater with each second. The sprinter in graph B is slowing down because the distance travelled each second becomes shorter. Notice that the graphs are both curved lines. When the speed increases, the graph curves upward. When the speed decreases, the graph curves downward.

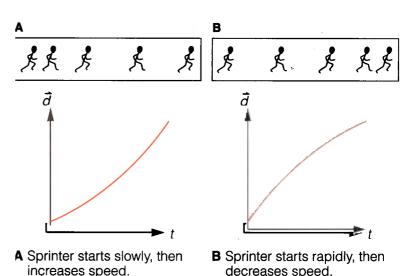


Figure 5.12 Position versus time graphs of accelerated motion are always curved.

By analyzing graphs of position versus time, you can determine whether the velocity is zero, constant, or changing. You can learn even more from a graph of velocity versus time, as shown in Figure 5.13. The position versus time graph in part A is a straight line indicating that the velocity is constant. The velocity versus time graph below shows this constant velocity. In part B, the position versus time graph is a curve, indicating that the velocity is changing. The velocity versus time graph below shows that the velocity is increasing. The fact that the velocity versus time graph is a straight line shows that the velocity increases uniformly. Earlier, you learned that a straight-line graph of position versus time (A) represents constant velocity. Similarly, a straight-line graph of velocity versus time shows that the acceleration is a constant. In part C, the position versus time graph curves in a downward direction. This direction is opposite to that of the curve in part B. The downward curve once again indicates that the velocity is changing. However, the direction of the curve means that the magnitude of the velocity is decreasing, as shown in the velocity versus time graph. The velocity versus time graph in part C shows that the velocity is decreasing uniformly, again indicating that the acceleration is constant.

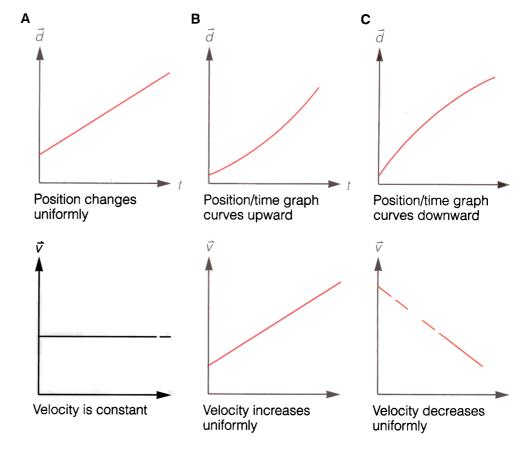


Figure 5.13 Each pair of graphs — above and below each other - represent the same motion. The upper graphs show position versus time and the lower graphs show velocity versus time. The lower graphs clearly show whether the velocity is constant, increasing, or decreasing. Therefore, these graphs reveal the acceleration of the object.

You might feel that you should use the term "deceleration" instead of "acceleration" for cases in which the magnitude of the velocity is decreasing. However, deceleration is not a scientific term. The correct term to describe the change in the motion represented by the graphs in part C is negative acceleration.

To practise combining information about position, velocity, and acceleration, complete the following activity.



Describing Motion

To get a clear understanding of an object's motion. especially if it is changing speed and/or direction. you need to determine the object's position and velocity of travel many times as it moves along. The more observations you make, the better you will be able to describe the details of the motion. Use this principle to study the changing motion of a runner during a short race.

Materials

athletic measuring tape (at least 50 m) as many electronic timers or digital watches as possible

whistle (for starter)



- 1. Choose one student to be the runner and one to be the starter. The rest of the students are timers.
- 2. Have the runner stretch and warm up while the rest of the class sets up a 50 m or 100 m straight racetrack. The timers must be equally spaced along the track and at least 2 m apart, and with one timer at the finish line. Practise synchronizing the start time before actually running the race.
- 3. Each timer must start his or her watch when the runner starts the race, and stop it when the runner passes his or her position. When everyone is ready, have the runner run the race.

- 4. Record your observations in a data table. Show the position of each timer (in metres. measured from the starting point). Show the time the runner took to reach each position.
- **5.** Draw a graph to show the position the runner reached at each different time.
- 6. Calculate the runner's average velocity for each segment of the race.
- 7. Make a graph of velocity versus time.

What Did You Find Out?

- 1. Answer the following questions:
 - (a) What is the runner's initial position and velocity?
 - (b) What is the runner's final position and velocity?
 - (c) Does the runner accelerate at any point in the race? Where? How can you recognize when the motion is accelerated?
 - (d) In what time intervals is the runner's velocity highest and lowest?
- 2. What information was easier to obtain from your graph than your data table? What information was easier to obtain from your data table?
- 3. How could you change your method of collecting data to get a more detailed picture of the runner's motion?
- **4.** How precise were your measurements?



Acceleration is the measure of the rate at which velocity changes. In free fall near Earth's surface, an object will accelerate at 9.81 m/s². This seems like a rather large rate of acceleration until you consider the following:

- a rat flea can jump with an acceleration of 2900 m/s²
- a squid can extend its tentacles at 330 m/s²
- a woodpecker's head stops with a negative acceleration of 100 m/s²
- a trout can start swimming with an acceleration of 40 m/s²
- a deer can jump at an acceleration of 16 m/s²
- a human can jump at an acceleration of 14 m/s²

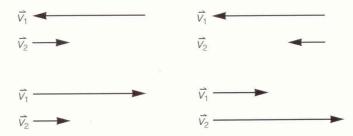


Section 5.1 Summary

In this section, you learned that scalar quantities have magnitude only while vector quantities have both magnitude and direction. You learned the meaning of the scalar quantities of distance, speed, and time. You also learned the meaning of the vector quantities of position, displacement, velocity, and acceleration. You developed skills in both calculating and graphing these quantities. In the next section, you will use some of these quantities to describe kinetic energy and learn how it is related to work.

Check Your Understanding

- 1. Explain the difference between distance and displacement.
- 2. How are position and displacement related to each other?
- 3. Explain the difference between "time" and "time interval."
- 4. Imagine that your house is 0.75 km[N] from school. Your friend's house is 1.6 km[S] from school. If it takes you 6.5 min to walk to your friend's house, what is your velocity?
- 5. A professional tennis player returns a serve, giving the ball a horizontal speed of 21.5 m/s. If her opponent is 21.7 m away, how long does it take for the ball to reach the opponent?
- 6. Apply Make a position versus time graph of the data in the table on the left. Draw a line of best fit and use the graph to determine the velocity.
- 7. Apply For each of the following combinations of initial and final velocity vectors, determine the direction of the acceleration. Explain how you determined the direction of the acceleration.



- 8. Apply Define acceleration. Give three different examples of the acceleration of an object.
- 9. Thinking Critically In your everyday experiences, you observe that everything that is moving eventually slows down and comes to a stop unless you continue to exert a force on it. For example, if you are riding your bicycle on a level road and stop pedalling, the bike will soon stop. A child on a swing will eventually slow down and stop unless someone pushes the child or she pumps the swing. You roll a ball along a horizontal surface and eventually it stops rolling. However, in this section, you read that if no forces are acting on a moving object, it will continue to move indefinitely. Explain why everyday experiences seem to contradict this principle of motion.

Position (m[E]) Time (s) 2.0 5.4 6.9 4.0 10.4 6.0 8.0 13.8 16.3 10.0